RG & EFT for nuclear forces

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- **Low momentum interactions:**
  Using the RG to simplify the nuclear force for many-body calculations.

- Application of chiral perturbation theory to nuclear systems:
  How to apply perturbation theory to a non-perturbative problem?

- Three-nucleon forces:
  importance of 3NF's for the quantitative description of (light) nuclei
  relation to low momentum interactions

References

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References that might be useful (I do not claim completeness)

- concerning low momentum interactions:

S.K. Bogner, T.T.S. Kuo, A. Schwenk, Phys. Rep. 386,1 (2003)
E. Epelbaoum, W. Glöckle, A. Krüger, Ulf-G. Meißner, Nucl. Phys. 645,413 (1999)

- Chiral Perturbation Theory in general:

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V. Bernard, N. Kaiser, Ulf-G. Meißner, Int. J. Mod. Phys. E 4, 193 (1995)

- Effective field theory for nuclear forces:

E. Epelbaum, nucl-th/0509032
P. F. Bedaque, U. van Kolck, Annu. Rev. Nucl. Part. Sci. 52, 339 (2002)
U. van Kolck, Prog. Part. Nucl. Phys. 43,337 (1999)

NN forces

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protons & neutrons (nucleons)

Schrödinger equation → Nuclei & nuclear reactions at low energy

nuclear potential

For the nucleon-nucleon (NN) system, potential **models** have been developed that describe the **NN data (6000 data)** below the pion production threshold almost perfectly.

There are two problems:

- the models usually include a very **strongly repulsive short-range** interaction that requires a special (non-systematic) treatment in many-body calculations.

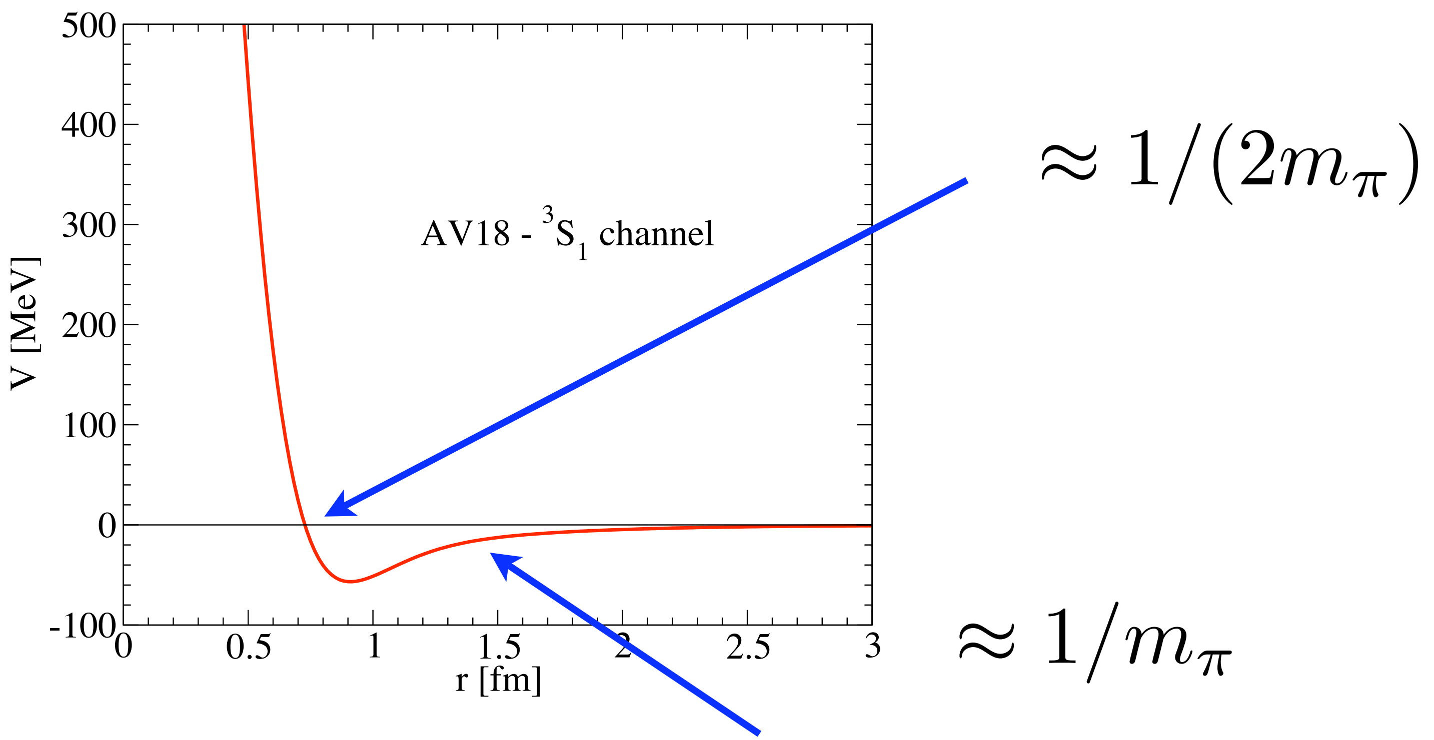
- the models are almost completely unconnected to QCD, especially there is not connection between the NN interaction and other strongly interacting processes. Even three-nucleon (3N) forces are models, which are not related to the NN force!

The following, I would like to address the first problem,

the second one will be addressed tomorrow.

NN forces

E.g. for a typical NN interaction: AV18 in the 3S1 channel

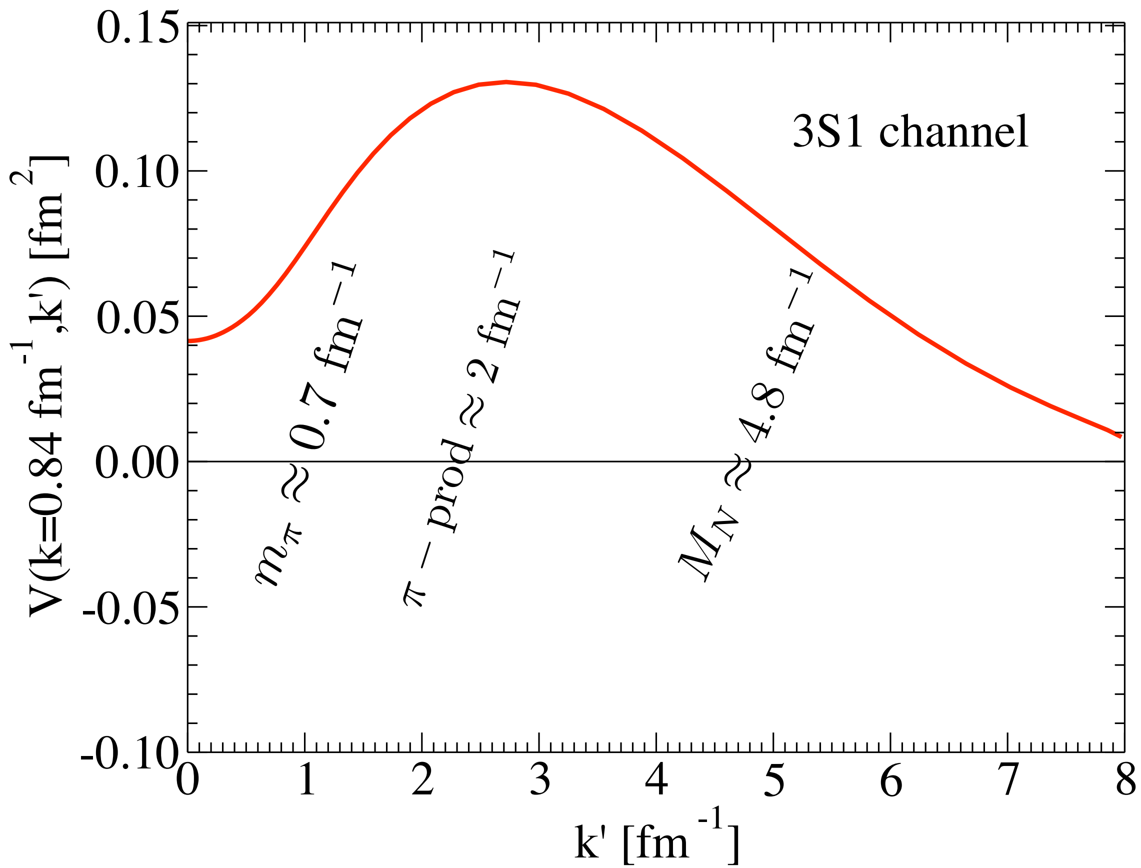


$$V_{1\pi}(\vec{r}) \propto [T(r)\ [3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2] + Y(r)\ \vec{\sigma}_1 \cdot \vec{\sigma}_2]$$

The long range part is given by the one-pion exchange
(including tensor-force, orbital angular momentum is not conserved).
The two-pion exchange is usually not included, instead there is the repulsive core !

NN forces

Fourier transforming into momentum space,
this translates to a high momentum tail ...



The interaction contains momentum components, which reach out to momenta higher than any data constraining the interaction, beyond pion production threshold

NN forces

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NN data, parameterized by **"phase shifts"**, can be obtained from the potential by a Lippmann-Schwinger (LS) equations.

Here, we use the LS equation for the "K-matrix"

$$K(p,p') = V(p,p') + \int_0^\infty dp''\; p''^2\; \frac{m\; V(p,p'')K(p'',p')}{mE - p''^2}$$

which is related to the S-matrix (for a coupled channel problem) by

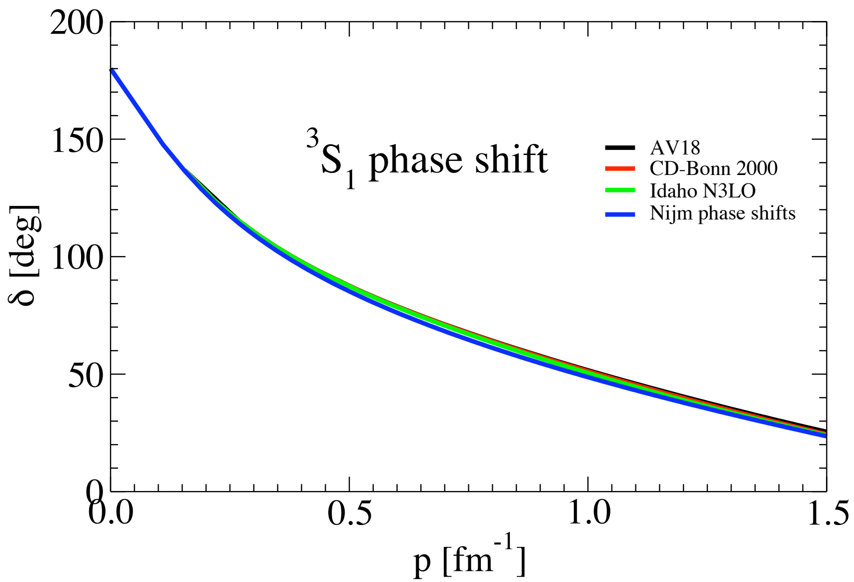
$$S = \begin{pmatrix} \cos 2\epsilon\; e^{2i\delta_1} & i\sin 2\epsilon\; e^{i(\delta_1+\delta_2)} \\ i\sin 2\epsilon\; e^{i(\delta_1+\delta_2)} & \cos 2\epsilon\; e^{2i\delta_2} \end{pmatrix} = (1 + i\pi \frac{m}{2}\; K)^{-1}\; (1 - i\pi \frac{m}{2}\; K)$$

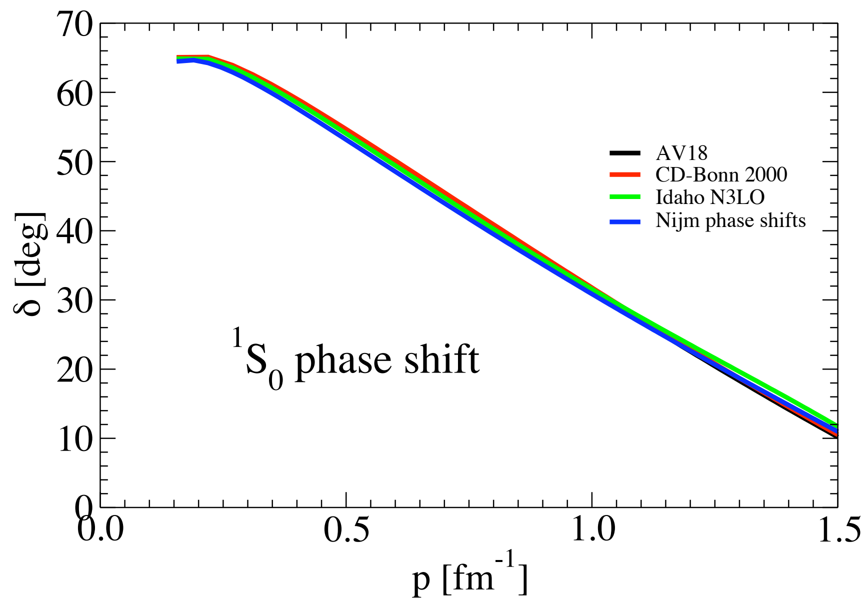
Note that for the numerical solution, the high momenta up to **40 fm^{-1}** are quite important!

NN forces

NN data → "phase shifts analysis" → phase shifts

Several NN interactions describe data & phase shifts perfectly.





But phase shift equivalence of NN interactions does not imply that the potentials are the same!

NN forces

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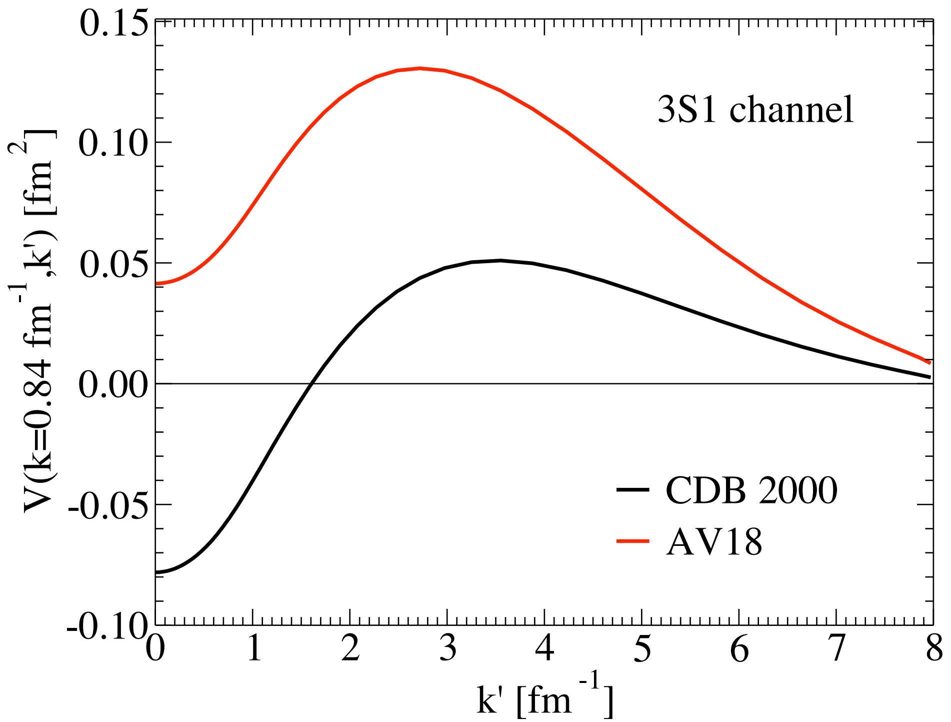
The non-observable potential matrix elements are extremely model dependent.



Obviously, the form of the interaction is not strongly constrained.

For solving a many-body problem, we need to remove the high momentum components.

In the following, we want to use an RG equation to remove the high momentum components of the interaction. What will happen to the different interactions?

RG for the NN force

In principle, it is possible to obtain an interaction without high momentum components by requiring that the half-off-shell K-matrix is preserved.

Considering a truncated LS equation

$$K(p,p') = V_\Lambda(p,p') + \int_0^\Lambda dp'' \; p''^2 \; \frac{m V_\Lambda(p,p'')\, K(p'',p')}{p'^2 - p''^2}$$

and $\quad \dfrac{d}{d\Lambda} K(p,p') = 0 \quad$ for the half-off-shell K-matrix.

One can obtain the RG equation

$$\frac{d}{d\Lambda} V_\Lambda(p,p') \equiv \beta(V_\Lambda,\Lambda) = -\frac{m\Lambda^2}{p'^2 - \Lambda^2} \; V_\Lambda(p,\Lambda) K(\Lambda,p')$$

Starting for a large Λ and evolving Λ down, one can obtain a low energy, but non-hermitian interaction $V_\Lambda(p,p')$.

For most purposes, the interaction is then hermitized (which is a small change).

This, however, is not the most efficient scheme to obtain the low energy potential.

P-Q block diagonalization

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The RG equation requires to run down $V_\Lambda(p,p')$ in small Λ steps. This is numerically demanding. It turns out that the **"Okubo transformation"** is equivalent and more efficient.

Divide full Hilbert space in small "model space P" and rest space Q.

unitary "Okubo" transformation

$$\tilde{H} = U\ H\ U^\dagger$$

$$H = \begin{pmatrix} PHP & PHQ \\ QHP & QHQ \end{pmatrix} \longrightarrow \tilde{H} = \begin{pmatrix} P\tilde{H}P & 0 \\ 0 & Q\tilde{H}Q \end{pmatrix}$$

If U was found, one could solve $P\tilde{H}P\tilde{\psi} = E\tilde{\psi}; \quad \tilde{\psi} = P\tilde{\psi}$

and find an **equivalent** solution in P space.

For low momentum P-space, we find again low momentum interaction $V_\Lambda(p,p')$

Used for other problems, like shell-model effective interactions, ...

P-Q block diagonalization

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Okubo showed that such a unitary transformation can be found.
It has the form (for the P-space matrix elements)

$$U P = (1+\omega)\ (1+\omega^\dagger \omega)^{-1/2} P$$

Where the operator ω has only matrix element going from P to Q space

$$\omega = Q\ \omega\ P$$

The operator ω is not unique, but for most purposes, one uses the lowest lying eigenstates $|n\rangle$ of the Hamiltonian for the definition.

$$Q\ |n\rangle = Q\ \omega\ P\ |n\rangle$$

completely defines ω.

And once we have found ω the effective Hamiltonian can be easily calculated.

Let's be a little bit more specific to understand the scheme also for a continuum of states.

P-Q block diagonalization

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The projection operators are defined

$$P = \int_{|\vec{p}|<\Lambda} d^3p \; |\vec{p}\rangle\langle\vec{p}| \qquad\qquad Q = \int_{|\vec{q}|\geq\Lambda} d^3q \; |\vec{q}\rangle\langle\vec{q}|$$

The eigenstates of H are given,e.g. , by the K-matrix

$$|\vec{p}^{\,(*)}\rangle = |\vec{p}\rangle + \frac{1}{E_p - H_0} \; K \; |\vec{p}\rangle \qquad \text{for} \quad |\vec{p}| < \Lambda$$

which we can insert into the defining equation for ω

$$Q\,|\vec{p}^{\,(*)}\rangle = Q \; \frac{1}{E_p - H_0} \; K \; |\vec{p}\rangle = Q \; \omega \; P\,|\vec{p}\rangle + Q \; \omega \; P \; \frac{1}{E_p - H_0} \; K \; |\vec{p}\rangle$$

which results in

$$\omega = \frac{1}{E_p - H_0} \; K - \omega \; P \; \frac{1}{E_p - H_0} \; K$$

Thus, we need to solve this Lippmann-Schwinger type equation to find the unitary transformation.

P-Q block diagonalization

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To finalize this formal part, let's have a brief look at the explicit form

$$\omega(\vec{q},\vec{p}) = \frac{1}{E_p - E_q}\, K(\vec{q},\vec{p}) - \int_{|\vec{p}\,'|<\Lambda} d^3p'\ \omega(\vec{q},\vec{p}\,')\ \frac{1}{E_p - E_{p'}}\, K(\vec{p}\,',\vec{p})$$

Because of propagator in the inhomogenity and the finite range in the integration, this equation is ill defined for momenta $|\vec{q}\,|, |\vec{p}\,| \approx \Lambda$.

Both issues can be solved by introducing a function, that suppresses K for $|\vec{q}\,|, |\vec{p}\,| \approx \Lambda$

Once the unitary transformed Hamiltonian has been found in this way numerically, one defines the effective interaction by

$$V_\Lambda^{\rm vlowk} = \tilde{H} - H_0$$

I remind you that in this way, one obtains the same "vlowk" as by using the RG equation.

The numerical outcome is most interesting. Let's have look ...

Model independence

Numerical evidence shows

"bare" starting interaction
is strongly model dependent

→

low momentum interaction "vlowk"
is **model independent**

In practice, this is important in two ways:

We will see that a state-of-the-art nuclear bound state calculation requires three-body interaction. The similarity of "vlowk" to chiral EFT interactions justifies a combination of an EFT three-nucleon force and "vlowk".

"vlowk" can replace an EFT interaction, when cutoff dependence of some observables need to be studied and one does not want to perform tedious refitting of EFT interactions.

Model independence

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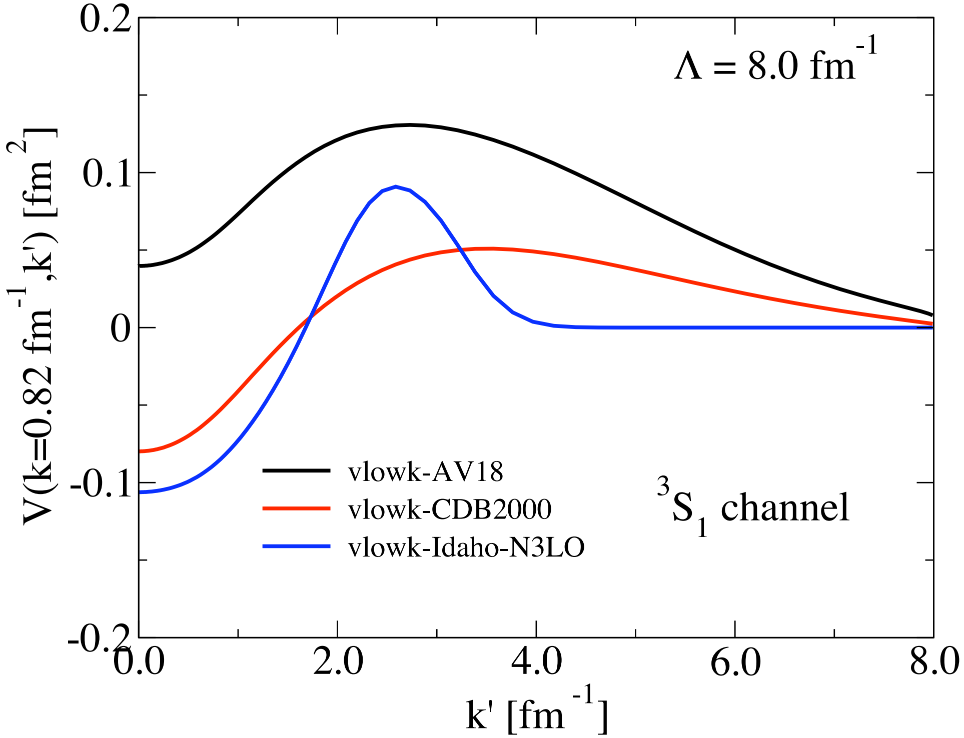
Let's have a look at a non-diagonal matrix element in the 3S1 channel for a large cutoff for which the "vlowk" interaction resembles the original forces.

I show vlowk for three examples, a very strongly repulsive local interaction, AV18,
a softer non-local interaction, CD-Bonn,
and an EFT based interaction, Idaho N3LO,
which is defined using a smooth cutoff of 2.5 fm^{-1}.

This is the result:

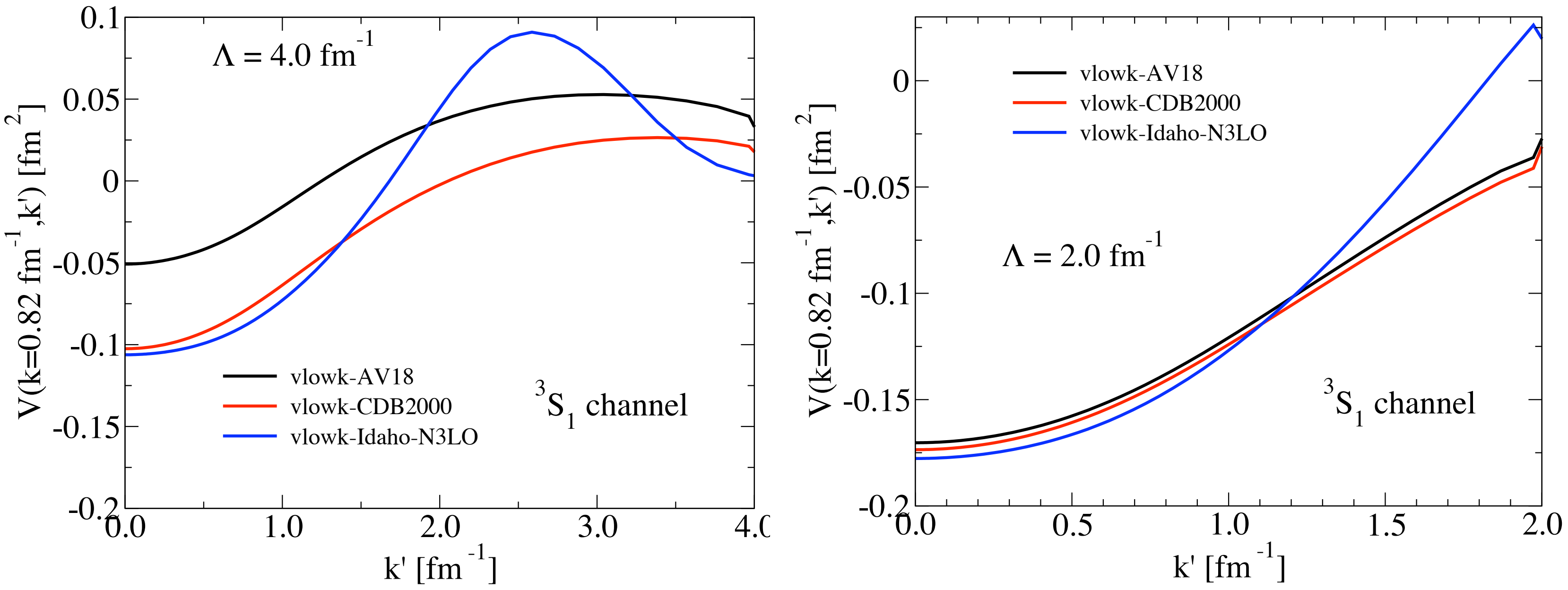


Model independence

Lowering the cutoff we observe that the interactions get closer and closer to each other.

CD-Bonn 2000 and AV18 are essentially collapsed to one interaction for $\Lambda = 2.0\ \mathrm{fm}^{-1}$

For the Idaho interaction, there are deviations, but these are rather mild.

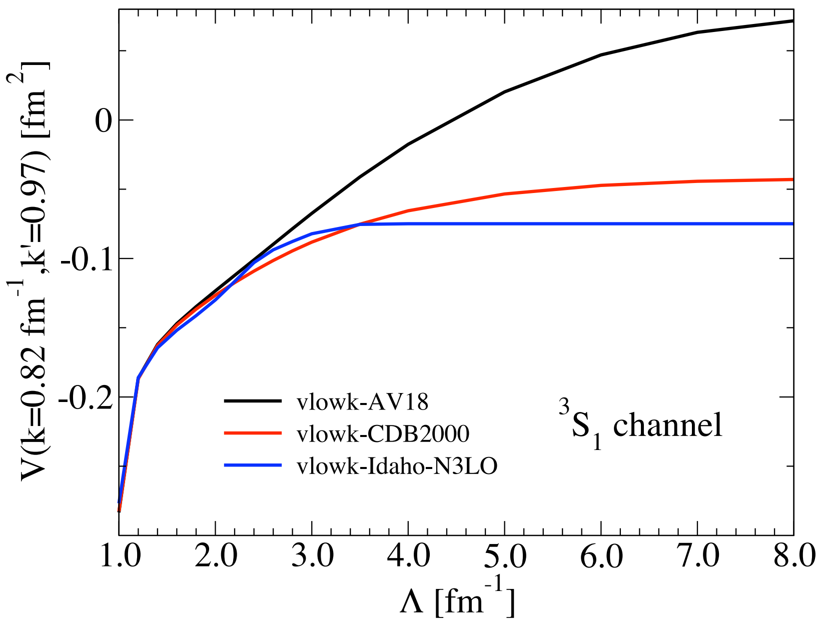


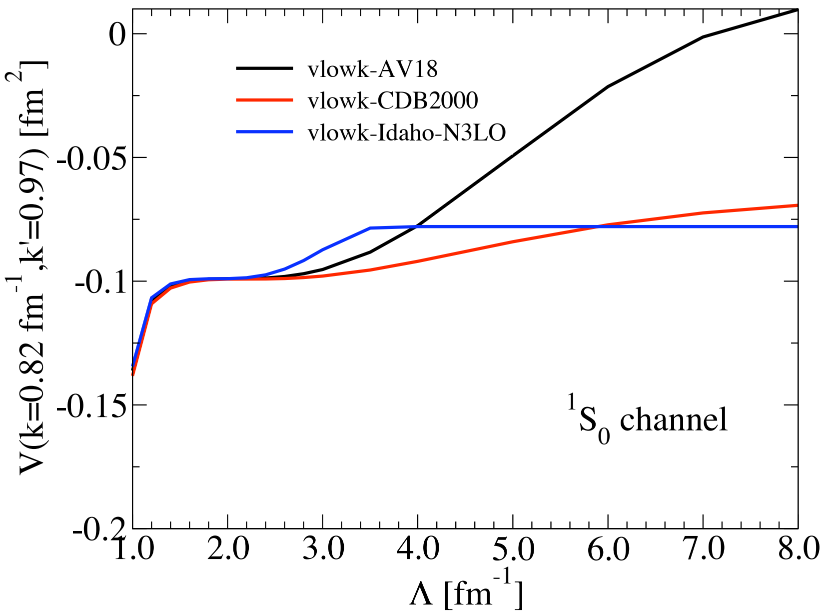
Model independence

This collapse of the "vlowk" is better seen, when one looks at a specific matrix element depending on the cutoff.

From here it is clear that the collapse happens around $\Lambda = 2.0 \text{ fm}^{-1}$ or a little bit higher

I note that the matrix element for 1S0 shows a clear plateau for these cutoffs.
Clearly, Idaho-N3LO is only renormalized for cutoffs smaller than its intrinsic one.





Model independence

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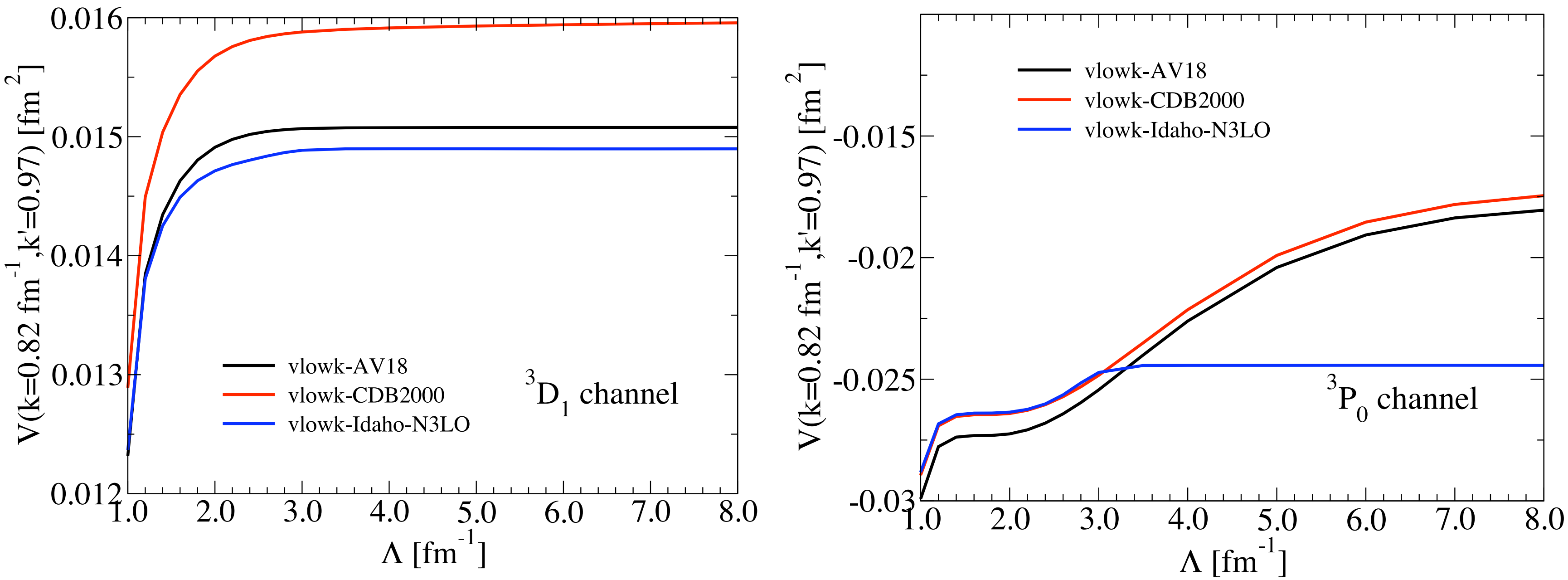
The p- and d-wave channels are anyway close to each other (Note the scale on the axis)

In summary, the different "vlowk" interactions get close to each other for cutoffs around $\Lambda = 2.0 \text{ fm}^{-1}$



Perturbativity

The second important property of "vlowk" is perturbativity.

Let's look at the LS equation using perturbation theory.

$$K = V + V\ G_0\ K = V + V\ G_0\ V + V\ G_0\ V\ G_0\ V + \ldots$$

This series would converge, if the eigenvalue spectrum of $G_0\ V$ contained only eigenvalues with magnitudes below 1.

Before showing the spectrum, let me add some notes on the spectrum of eigenvalues

a) Since $G_0\ V$ depends on the energy, the spectrum is also dependent on the energy. The magnitude of the eigenvalues will be largest for small E.
(I chose E=-0.3 MeV)

b) positive eigenvalues close to one will lead to bound states, when the potential strength is slightly increased

c) large negative eigenvalues correspond to bound states of (-V). They are generated by a repulsive core.

Perturbativity

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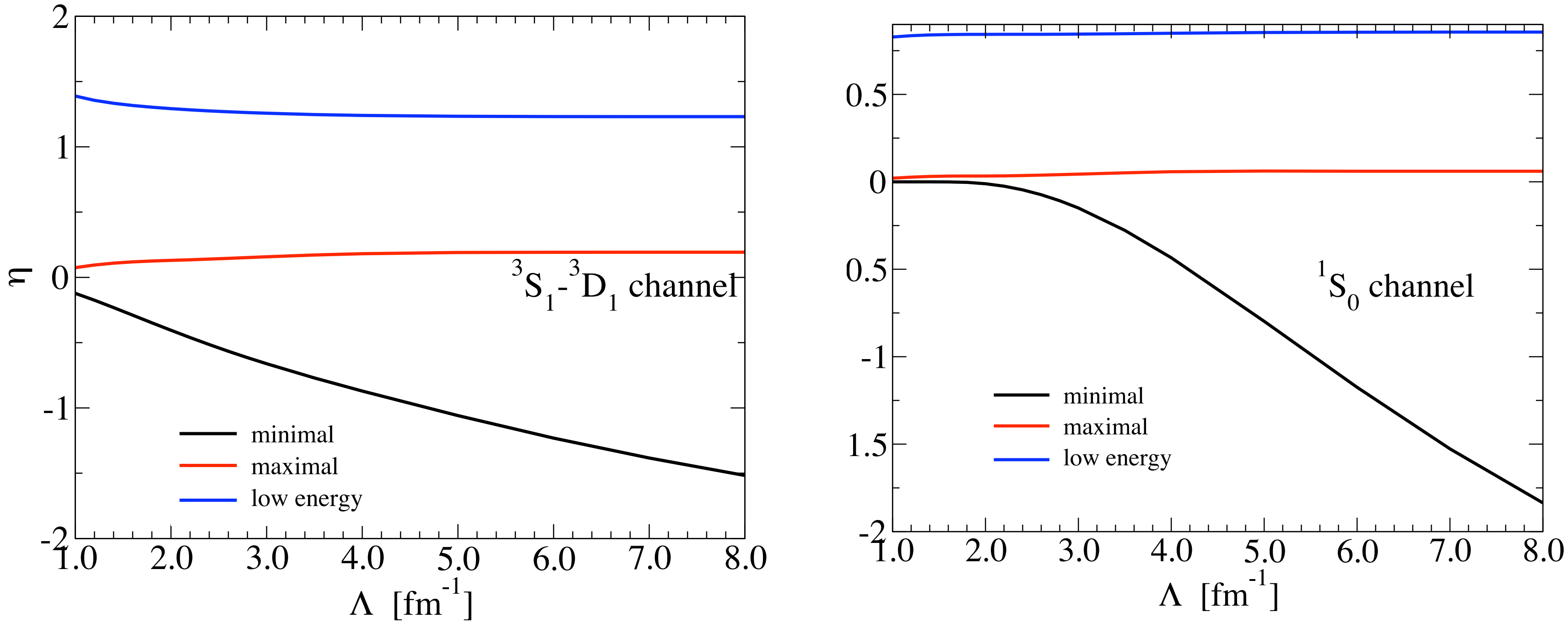
The spectrum of $G_0\ V$ strongly depends on Λ .

In the s-waves, where we have physical (virtual) bound states, exactly one eigenvalue is independent of Λ . This is the only one related to low energy physics!

If you look at the remaining ones, you see that they are strongly suppressed for small cutoffs. (Shown is the largest positive and negative eigenvalue in magnitude).

This is an important feature for many-body calculations.



Perturbativity

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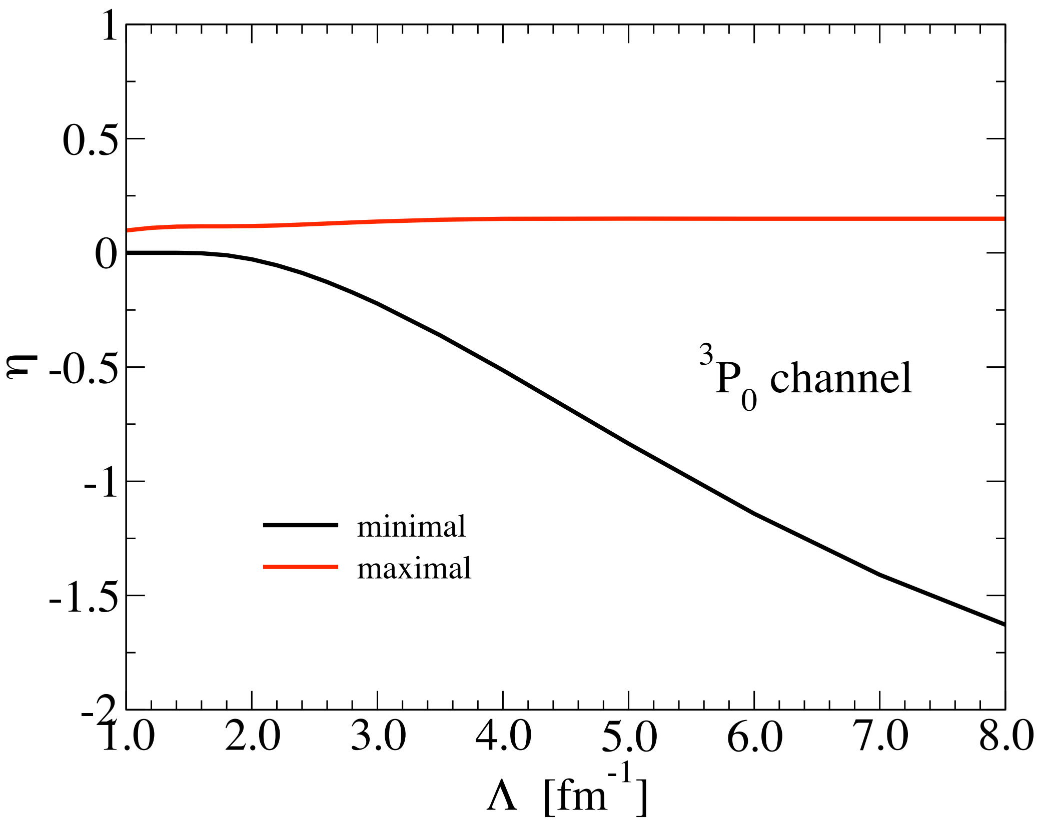
The spectrum of $G_0\ V$ strongly depends on Λ .

As an example for higher partial waves, I choose the 3P0 channel.
This time, no Λ independent eigenvalue is large! (no physical bound state!)
Again, for small Λ , the eigenvalues are small and the interaction is perturbative.



"vlowk" beyond phase shifts

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What did we gain by defining a "vlowk" interaction?

Unitary transformation in NN system → phase shift equivalent, model independent, but Λ dependent interaction

- All few-nucleon observables (starting from ^{3}H, ^{4}He,...) could be cutoff dependent!
- Processes involving photons, neutrinos, ... could be cutoff dependent.

"vlowk" is useful only if this is not the case for low energy observables!

Only an explicit calculation of the cutoff dependence can show whether this is indeed true for a specific observable.

Of course, none of these observables will be strictly cutoff independent.
Therefore, I need to define what I understand as mildly cutoff dependent.

"vlowk" beyond phase shifts

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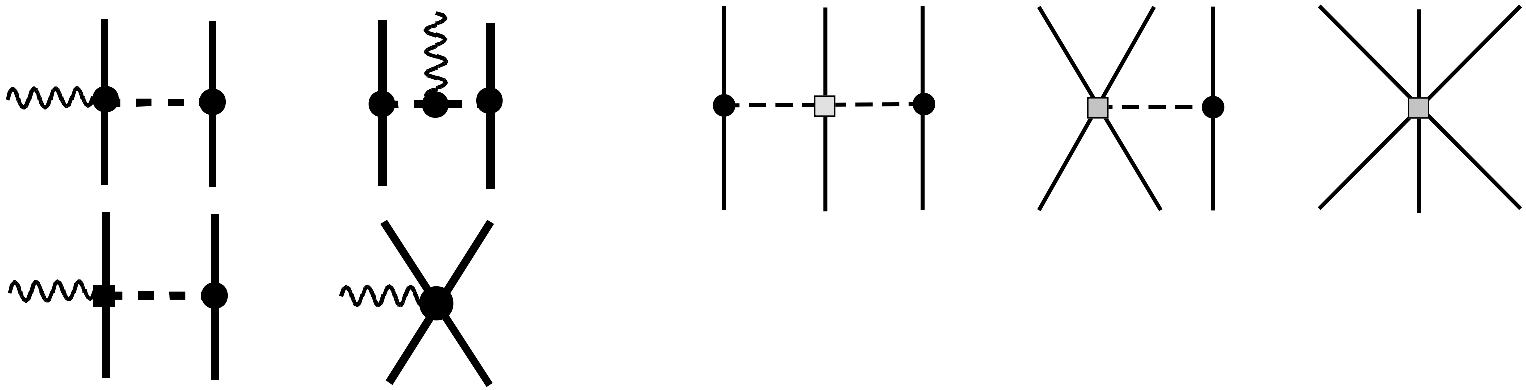
All these observables will generally have the leading contributions and higher order contributions:

- Few-nucleon observables have contributions from three- and higher-order forces (3NF's etc.)
- EM processes are influenced by, e.g. , meson-exchange currents (MEC's)

As long as I do not take these additional dynamics into account,
I will fail to describe the experiment.

Obviously, any cutoff dependence smaller or similar to the discrepancy to the experiment is mild.



"vlowk" beyond phase shifts

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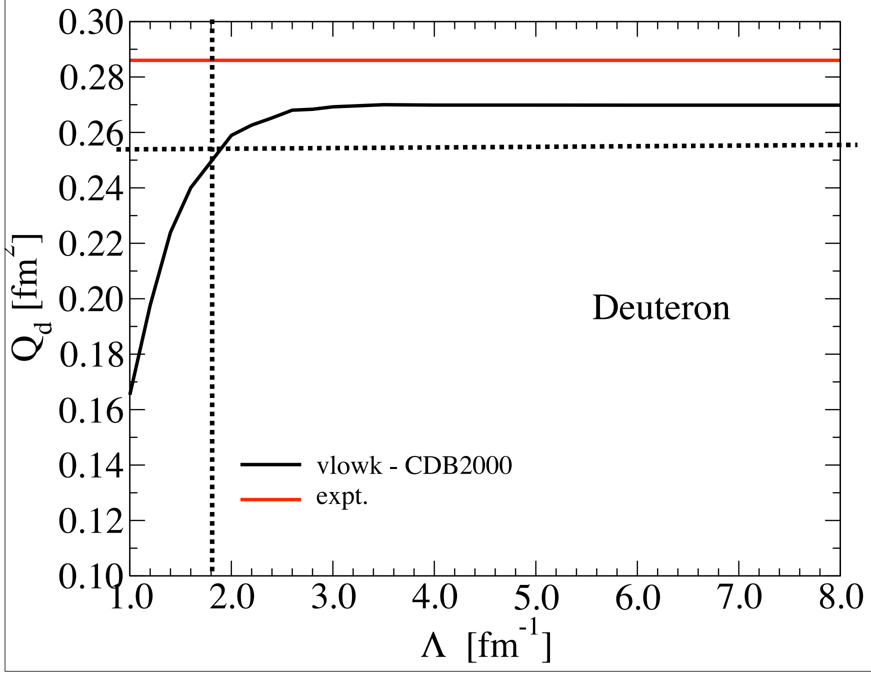
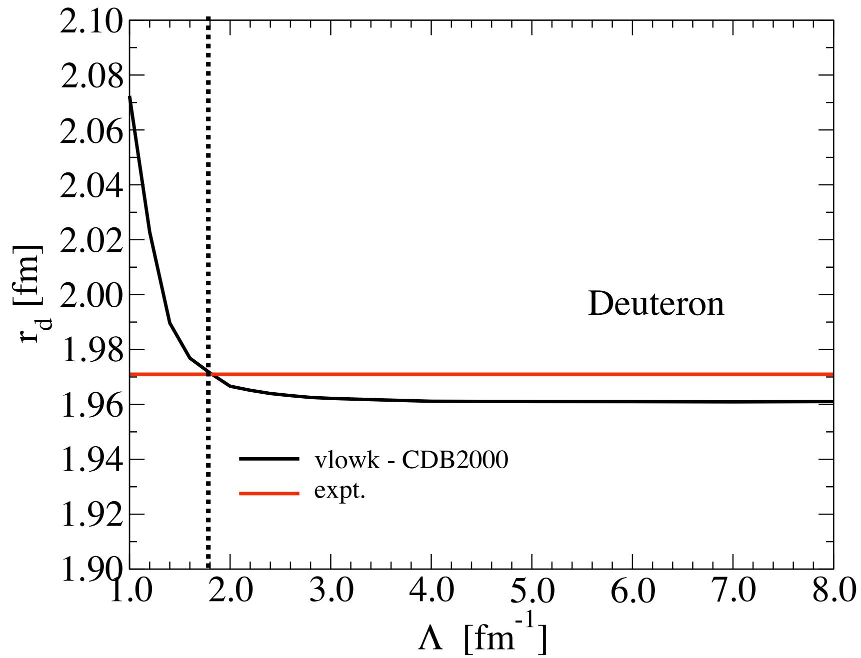
First and simple tests are the quadrupol moment and radius of the deuteron.

The explicit calculations shows that for both cases the variation with the cutoff is smaller or equal the deviation from the experiment as long as the cutoff is larger than something around 1.8 fm^{-1}. (this is an order of magnitude estimate!)

This confirms that even for cutoffs as small as 1.8 fm^{-1} the **relevant** dynamics are included!





"vlowk" beyond phase shifts

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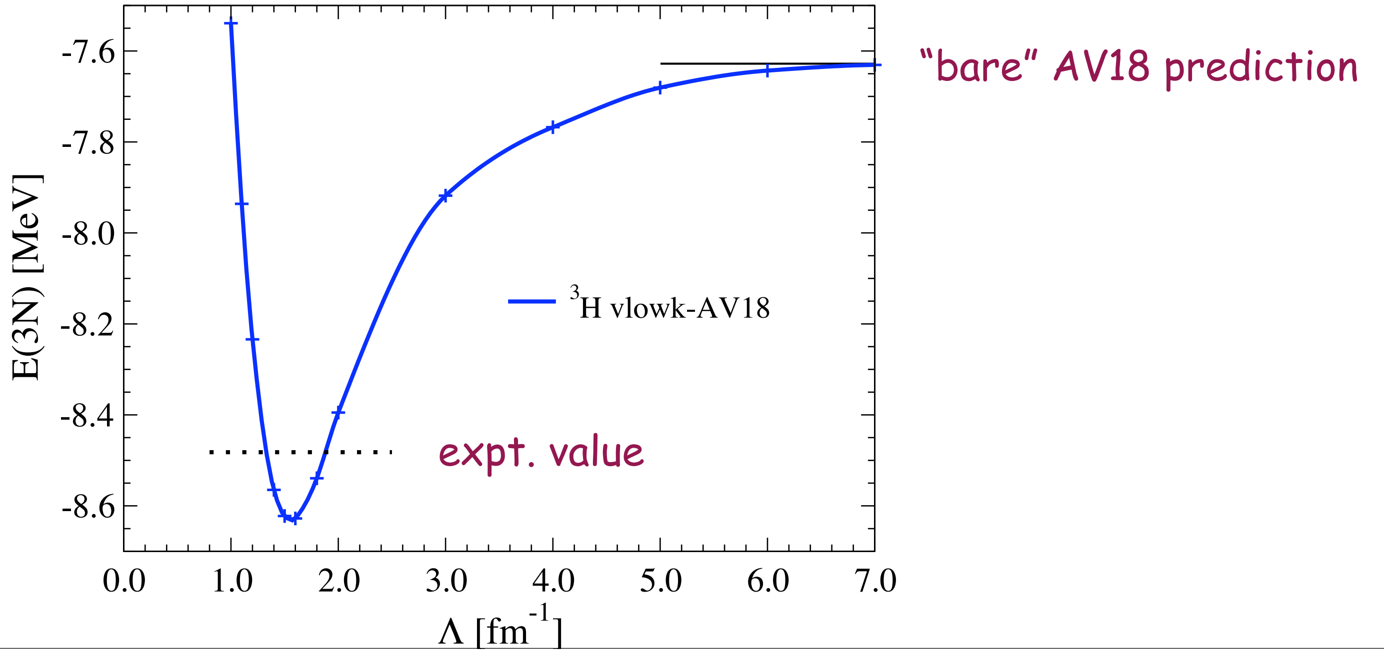
Binding energies for light nuclei are also good test cases.

For 3H the deviation of predictions based on NN forces usually deviate from the experiment by 500-800 keV.

The explicit calculations shows that for both cases the variation with the cutoff is smaller or equal 1 MeV for all cutoffs above 1.0 fm^{-1}.

This again confirms that the **relevant** dynamics are included!



Summary "vlowk"

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The prediction of nuclear observables is difficult.

Technically, a lot of these difficulties are due to a very strongly repulsive interaction at short distances, which generates a high momentum tail in the forces.

An RG inspired method can be used to obtain "vlowk"

1) these high momenta can be integrated out

2) the resulting interaction is model independent
and agrees for small momenta with EFT inspired interactions

3) the resulting interaction can be treated perturbatively, except where we have a low energy (virtual) bound state.

4) there is not significant cutoff dependence for low energy observables down to cutoffs below 2.0 fm^{-1}.

Insights from chiral EFT will be important to complete a "vlowk" based nuclear interaction, e.g. 3NF's are required for a quantitative description of binding energies.